

Interpreting VARs

Module 12 · Impulse responses are differences from a baseline

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Act I · One different shock

What changes relative to the path we would otherwise have seen?

Where we are: the same system, a new question

C02 · MODULE 11’S CONNECTED AUTOREGRESSIVE SYSTEM

$$\mathbf{Y}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{Y}_{t-1} + \boldsymbol{\epsilon}_t$$

Mixing-board channel	Module 12
Own and cross-variable memory	Keep the VAR dynamics
Current connections	State the shock’s identification
New question	Trace one extra shock through the system

> The object is a path of deviations from baseline.

Which coefficient tells the whole story?

C03 · RECALL MODULE 11

Direct structural cross-lag	Reduced-form cross-lag
x_{t-1} in the y_t equation: 0	x_{t-1} in the y_t equation: 8/35

The second coefficient combines connections in the system.

> What happens now, next period, and after that when one disturbance changes?

Change one shock, once

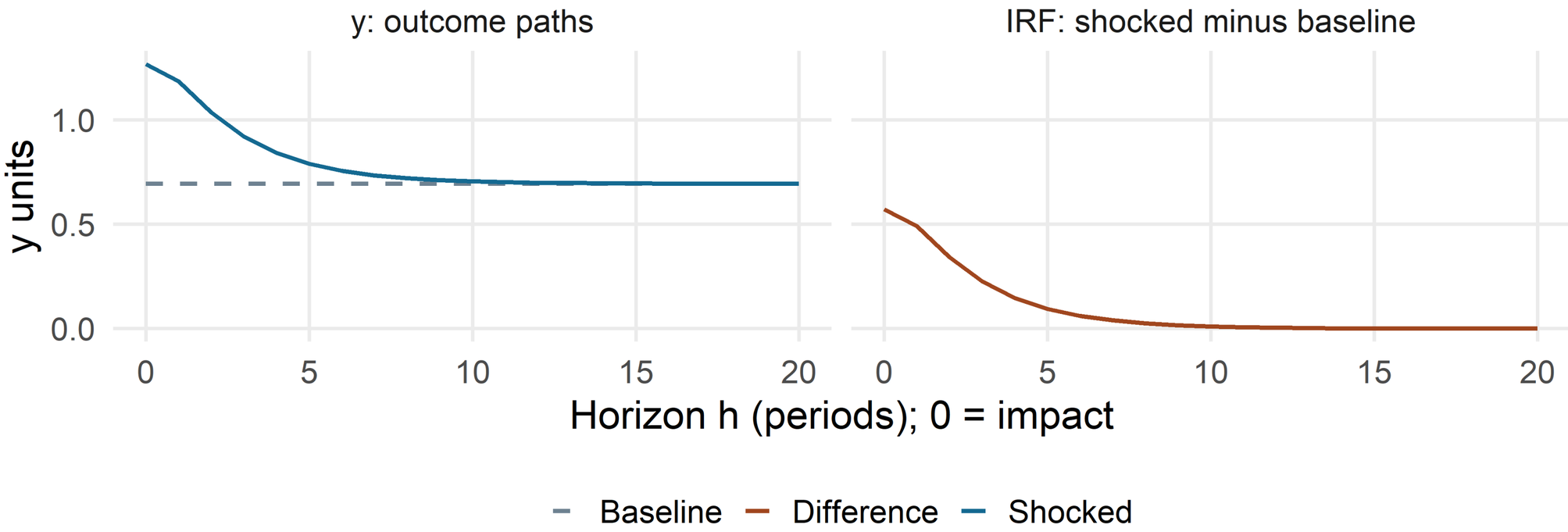
Before t	At t	After t
Same history	Add one specified shock to one copy	Same subsequent innovations

The other copy is the baseline: what happens without that extra shock.

> Does changing a disturbance mean holding the observed variable at a new level forever?

Two paths—and their difference

C05 · KNOWN MODULE 11 SYSTEM · DEFAULT: +1 X STRUCTURAL-SHOCK SD · PERIODS



The IRF is what remains after subtraction

C06 · CARRY THIS IDENTITY THROUGH EVERY FIGURE

IRF = shocked path − baseline path

baseline + IRF = shocked path

$$\mathbf{r}_h^{(j)}(\delta) = \mathbf{Y}_{t+h}^{\text{shock}} - \mathbf{Y}_{t+h}^{\text{base}}$$

> The IRF alone gives the deviation. It does not reveal the baseline.

A funds-rate response is relative

C07 · HYPOTHETICAL ILLUSTRATION · DEFINED INFLATION SHOCK · H = 4

Suppose the IRF of the funds rate is +0.5 percentage point.

Baseline rate	IRF	Rate with the extra shock
2.0%	+0.5 pp	2.5%
4.0%	+0.5 pp	4.5%

> The same response is compatible with different rate levels.

“At rest” is a useful starting picture

C08 · THE COMPARISON CAN ALSO HAVE A MOVING BASELINE

At the system mean, with zero later innovations, our baseline is flat.

With a different history or common later innovations, it can move.

> Could the rate rise over time and still have a negative response to the extra shock?

Yes—if it rises less than the baseline would have risen.

Act II · Today’s impact becomes tomorrow’s history

Where does the response come from when there is no new pulse?

Return to the system we know

$$
\begin{aligned}
y_t &= 0.2 + 0.5x_t + 0.3y_{t-1} + 0x_{t-1} + \eta_{y,t}, \\
x_t &= 0.1 + 0.25y_t + 0.1y_{t-1} + 0.4x_{t-1} + \eta_{x,t}.
\end{aligned}
$$

Add one unit to $\eta_{x,t}$, holding the other disturbance fixed.

> Here $\mathrm{Var}\left(\boldsymbol{\eta}_t\right) = \mathbf{I}$: one unit also equals one structural-shock SD.

Impact is a simultaneous solution

C10 · SUBTRACT THE BASELINE EQUATIONS AT H = 0

$$r_{y,0} = 0.5r_{x,0}, \qquad r_{x,0} = 0.25r_{y,0} + 1$$

$$\mathbf{r}_0 = \begin{pmatrix} 4/7 \\ 8/7 \end{pmatrix} \approx \begin{pmatrix} 0.5714 \\ 1.1429 \end{pmatrix}$$

> Why does y move even though its own structural disturbance did not change?

One period later, the pulse is gone

C11 · YESTERDAY'S RESPONSES ENTER TODAY'S EQUATIONS

$$r_{y,1} = \frac{2}{5} r_{y,0} + \frac{8}{35} r_{x,0} = \frac{24}{49} \approx 0.4898,$$
$$r_{x,1} = \frac{1}{5} r_{y,0} + \frac{16}{35} r_{x,0} = \frac{156}{245} \approx 0.6367.$$

There is no new +1. What keeps the response alive?

The same rule applies again

C12 · CARRY THE PREVIOUS RESPONSE FORWARD

Horizon h	y deviation	x deviation
0	0.5714	1.1429
1	0.4898	0.6367
2	0.3415	0.3890

$$\begin{pmatrix} r_{y,2} \\ r_{x,2} \end{pmatrix} = \begin{pmatrix} 2/5 & 8/35 \\ 1/5 & 16/35 \end{pmatrix} \begin{pmatrix} r_{y,1} \\ r_{x,1} \end{pmatrix}$$

Matrices record the rule compactly

C13 · IMPACT FIRST; PROPAGATION AFTER THAT

$$\mathbf{r}_0^{(j)}(\delta) = \mathbf{B}^{-1}\mathbf{q}_j\delta$$

$$\mathbf{r}_{h+1}^{(j)}(\delta) = \mathbf{A}_1\mathbf{r}_h^{(j)}(\delta)$$

$$\mathbf{r}_h^{(j)}(\delta) = \underbrace{\mathbf{A}_1^h}_{\text{propagation}}\underbrace{\mathbf{B}^{-1}\mathbf{q}_j\delta}_{\text{impact}}$$

The moving-average view collects the paths

C14 · RECALL THE UNIVARIATE AR-TO-MA REPRESENTATION

$$\mathbf{Y}_t - \boldsymbol{\mu} = \sum_{h=0}^{\infty} \boldsymbol{\Phi}_h \boldsymbol{\epsilon}_{t-h}, \qquad \boldsymbol{\Phi}_h = \mathbf{A}_1^h$$

For standardized structural shocks:

$$\boldsymbol{\Theta}_h = \boldsymbol{\Phi}_h \mathbf{B}^{-1} \boldsymbol{\Omega}^{1/2}$$

Column j of $\boldsymbol{\Theta}_h$ gives the response to one structural-shock SD.

Let R repeat the known rule

C15 · LIVE CODE · SAME KNOWN STRUCTURAL SHOCK

```r
r_live <- matrix(0, H + 1, 2)
r_live[1, ] <- B_inverse %*% c(0, 1)  # h = 0
for (h in 1:H) {
  r_live[h + 1, ] <- A1 %*% r_live[h, ]
}
```

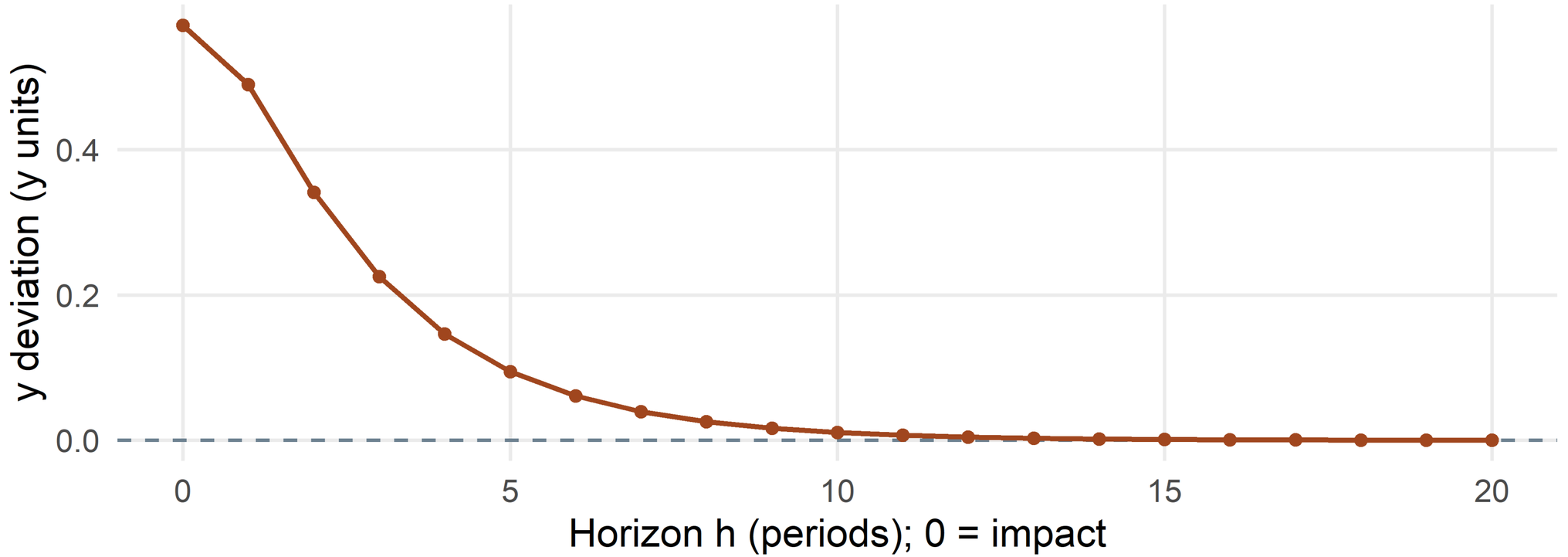
> Which line changes if we choose another shock? Which line propagates it?

Act III · Read the experiment, then the curve

A curve needs a shock, a scale, a clock—and a baseline interpretation.

Read the caption before the shape

C16 · Y RESPONSE TO +1 UNIT OF ETA-X · PERIODS · NONCUMULATIVE



> Every point answers: how far from baseline at this horizon?

One unit, one SD, and 100 basis points

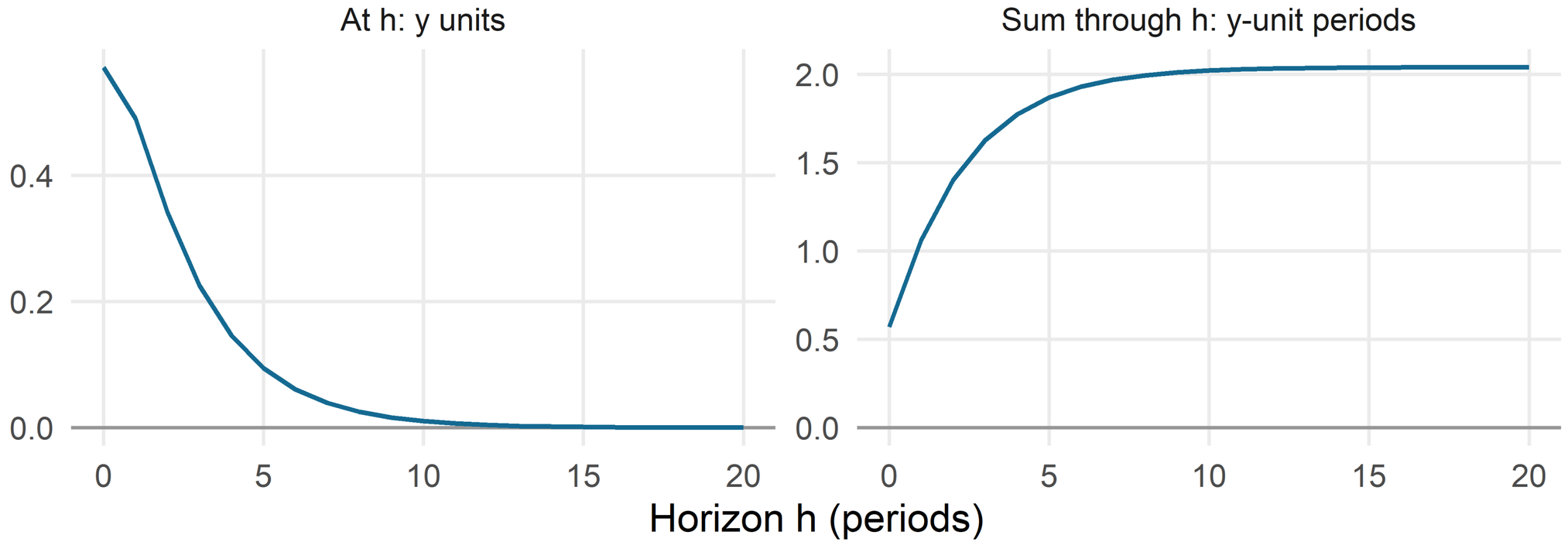
C17 · DIFFERENT LABELS CAN DEFINE DIFFERENT EXPERIMENTS

Shock label	What is fixed?
One unit	The disturbance’s numerical increment
One shock SD	Its scale under the stated identification
−100 bp policy impact	A rate decrease of one percentage point

> Does one SD of a reduced innovation equal one SD of a structural shock?

At a horizon—or accumulated through it?

C18 · SAME +1-UNIT X STRUCTURAL SHOCK · Y RESPONSE



$$R_{y,h} = \sum_{s=0}^{h} r_{y,s}$$

The transformation decides the units

C19 · CHECK WHAT ENTERED THE MODEL BEFORE USING CUMULATIVE = TRUE

Modeled variable	Its point IRF	Its cumulative IRF
Level y	Level-unit deviation	Sum of level-unit deviations
Difference Δy	Difference in changes	Level deviation, if initial levels match
Log difference $\Delta \log y$	Difference in log growth	Log-level deviation

> A response in level units is still a deviation, not the actual level.

What does linearity promise?

C20 · PREDICTIONS FROM THE MODEL

$$\mathbf{r}_h^{(j)}(-\delta) = -\mathbf{r}_h^{(j)}(\delta), \qquad \mathbf{r}_h^{(j)}(2\delta) = 2\mathbf{r}_h^{(j)}(\delta)$$

> Before changing the widget: what should −1 SD and +2 SD do to the entire path?

Reverse the sign. Double the deviation at every horizon.

Stability erases the extra shock’s footprint

C21 · A STABLE VAR FORGETS A ONE-TIME PULSE

$$\mathbf{r}_h^{(j)}(\delta) \to \mathbf{0}$$

The shocked and baseline paths converge toward one another.

The observed series may keep moving as other shocks arrive.

> If the cumulative response stays positive, must the point response stay positive forever?

Act IV · Identification changes the shock

Can the same reduced system support different response stories?

The covariance needs an economic interpretation

C22 · HIDE THE STRUCTURAL TRUTH FOR A MOMENT

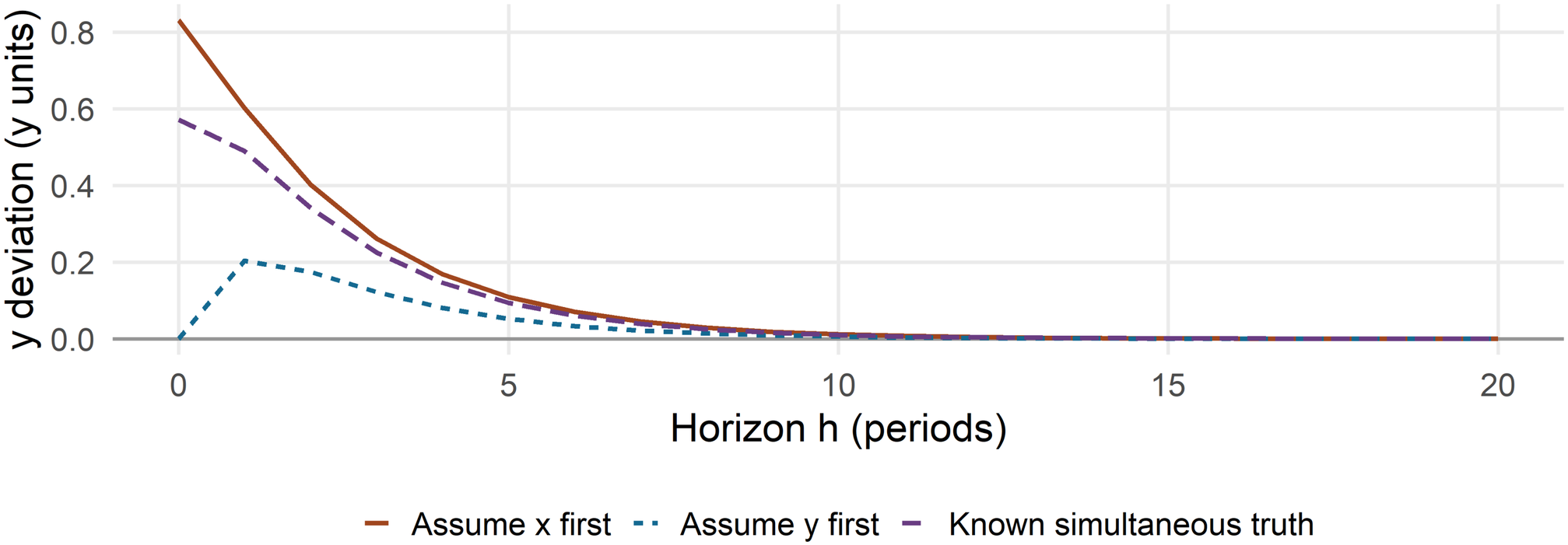
$$\boldsymbol{\epsilon}_t = \mathbf{C}\mathbf{z}_t, \qquad \text{Var}\left(\mathbf{z}_t\right) = \mathbf{I}, \qquad \boldsymbol{\Sigma} = \mathbf{C}\mathbf{C}'$$

$$\text{response matrices} = \boldsymbol{\Phi}_h \mathbf{C}$$

> A covariance factorization supplies a map. Restrictions give that map its economic meaning.

Change the restriction; keep the reduced system

C23 · DEFAULT: +1 SD OF EACH MODEL'S X-ASSOCIATED SHOCK



Both recursive restrictions are false here

C24 · WE KNOW BECAUSE WE BUILT THE GENERATOR

Assumption	Excluded current connection	Generating coefficient
y first	$x_t \to y_t$	0.50
x first	$y_t \to x_t$	0.25

Both still reproduce the same reduced-form information.

> Would an enormous sample make either zero restriction true?

What `ortho` asks R to compute

C25 · FOR AN OBJECT RETURNED BY VAR()

Option	Impact experiment	Propagation
`ortho = FALSE`	One-unit reduced innovation; other coordinates zero	$\widehat{\mathbf{\Phi}_h}$
`ortho = TRUE`	One-SD recursive shock, in the fitted order	$\widehat{\mathbf{\Phi}_h}\widehat{\mathbf{P}}$

$$\widehat{\mathbf{\Sigma}} = \widehat{\mathbf{P}}\widehat{\mathbf{P}}'$$

> Choose the experiment you intend. The switch does not validate an economic shock.

Make the choice explicit in code

C26 · LIVE R · MODULE 11 SAMPLE · N = 2000 · SEED 637611

```r
irf_reduced <- irf(fit, impulse = "x", response = c("y", "x"),
  n.ahead = H, ortho = FALSE, cumulative = FALSE, boot = FALSE)
irf_recursive <- irf(fit, impulse = "x", response = c("y", "x"),
  n.ahead = H, ortho = TRUE, cumulative = FALSE, boot = FALSE)
```

> An identified `SVAR()` object already carries its impact map.

`ortho = FALSE` is not a generalized IRF. Optional notes explain that separate object.

Selecting the impulse does not reorder the model

C27 · THE VARIABLE ORDER BELONGS TO THE FITTED SYSTEM

```
fit_x_first <- VAR(var_data[, c("x", "y")], p = 1, type = "const")
irf_x_first <- irf(fit_x_first, impulse = "x", response = c("y", "x"),
  n.ahead = H, ortho = TRUE, cumulative = FALSE, boot = FALSE)
```

> What changed: the reduced forecasting system, or the shock’s identifying assumption?

Act V · A curve is an estimate

How much of this apparent response is precisely estimated?

Bootstrap the response, not just one coefficient

C28 · REPEAT THE FITTED-MODEL EXPERIMENT

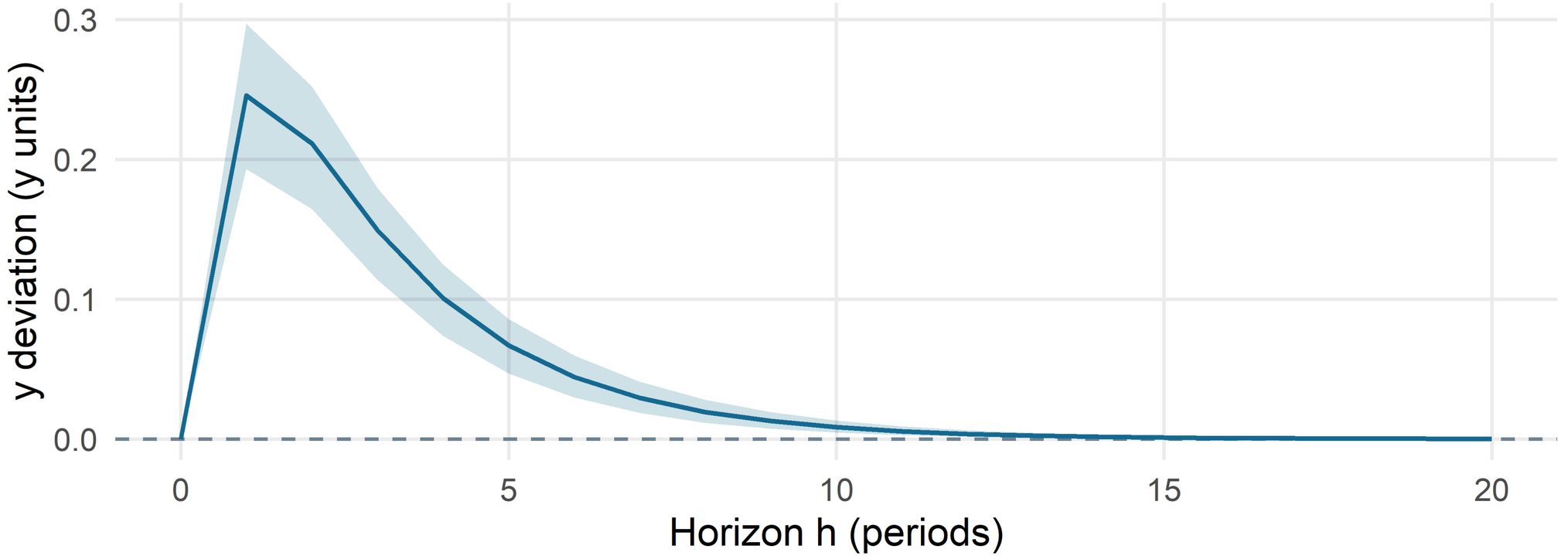
Pseudo-data → refit VAR → recompute IRF

At each horizon, compare the responses across replications.

Sample residuals as whole vectors across equations.

Read size and precision separately

C29 · ESTIMATED Y RESPONSE · ONE-SD Y-FIRST X SHOCK · 95% POINTWISE BAND



1,000 residual-vector bootstrap replications · seed 637612 · percentile intervals

Pointwise bands concern a selected horizon

C30 · CONNECTING INTERVALS DOES NOT CHANGE THEIR COVERAGE CLAIM

At a specified horizon: interpret its interval under the procedure.

Across the whole path: simultaneous coverage needs a different construction.

> A response band is not a prediction interval for the future observed level.

Narrow bands do not validate the restriction

C31 · SAMPLING UNCERTAINTY IS CONDITIONAL ON A MODEL

The chosen variables, lag specification and identifying assumptions remain part of the claim.

> If both recursive curves were estimated almost perfectly, which identification problem would remain?

> Precision about a response under an assumption is not evidence that the assumption is true.

Act VI · Read Bianco as an applied researcher

What does a policy easing do to credit creation and destruction?

Net credit can hide gross movements

C32 · BIANCO (2021), MONETARY POLICY AND CREDIT FLOWS

Some firms expand debt while others contract it.

The net change can conceal both movements.

> Could monetary policy affect credit creation and destruction differently?

Know what the model measures

C33 · QUARTERLY COMPUSTAT · 1974Q1–2017Q1

Object	Meaning
Credit creation	Debt-weighted positive firm debt growth
Credit destruction	Debt-weighted absolute negative firm debt growth
Macro controls	GDP, unemployment, consumer prices
Policy rate	Effective funds rate; Wu–Xia substitute at the lower bound

> A rise in destruction means more credit contraction.

Translate the ordering into an impact claim

C34 · ROWS: RESPONSES · COLUMNS: SHOCKS · SAME ORDERED LABELS

1. GDP
2. Unemployment
3. Consumer prices
4. Policy rate
5. Credit destruction
6. Credit creation

$$\mathbf{C} = \begin{pmatrix} * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \end{pmatrix}$$

> Which variables may respond on impact to the policy shock?

The paper also restricts a lag coefficient

C35 · A SEPARATE RESTRICTION FROM CONTEMPORANEOUS ORDERING

The first lag of the policy rate is excluded from the price equation.

The restricted system is estimated by iterated SUR.

> An unrestricted VAR plus `ortho = TRUE` does not reproduce the full specification.

Read Figure 4’s experiment first

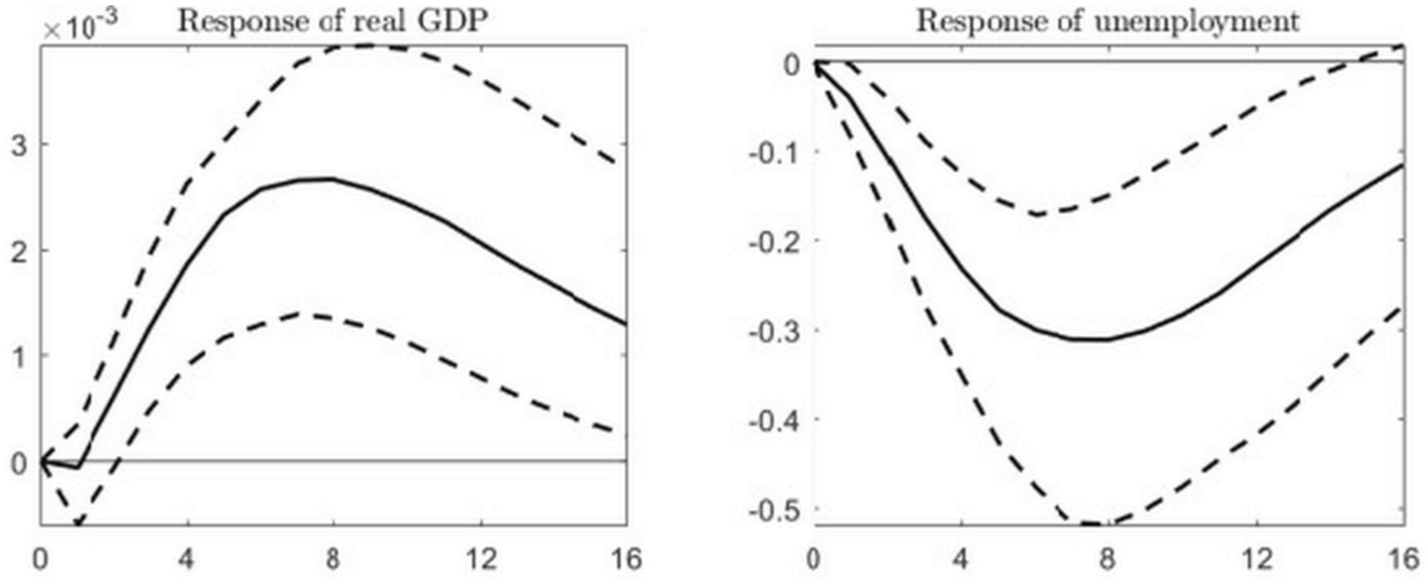
C36 · THE CAPTION IS PART OF THE RESULT

Feature	Figure 4
Shock	One-time unexpected −100 bp policy easing
Clock	Quarters; 0 = impact
Dynamics	Two lags, selected by BIC
Bands	68%, mean-bias-corrected residual wild bootstrap

> Every panel is a departure from the path without the added policy shock.

Output and unemployment respond over time

C37 · −100 BP POLICY EASING · QUARTERS · 68% BANDS



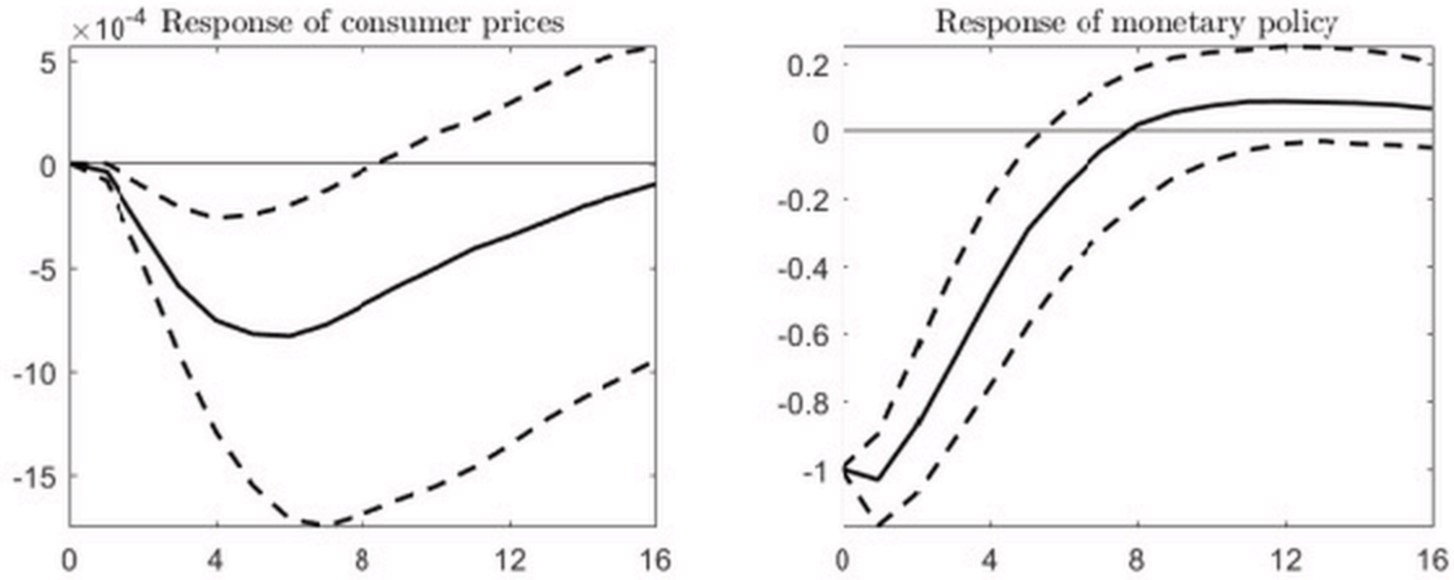
Published GDP and unemployment IRFs. Original axes and bands retained.

> Describe direction, timing and persistence relative to baseline.

Bianco (2021), Figure 4, p. 12 · Qualitative reading; retain source scaling.

The policy response is not a rate level

C38 · CONSUMER PRICES AND POLICY RATE · −100 BP EASING · QUARTERS · 68% BANDS



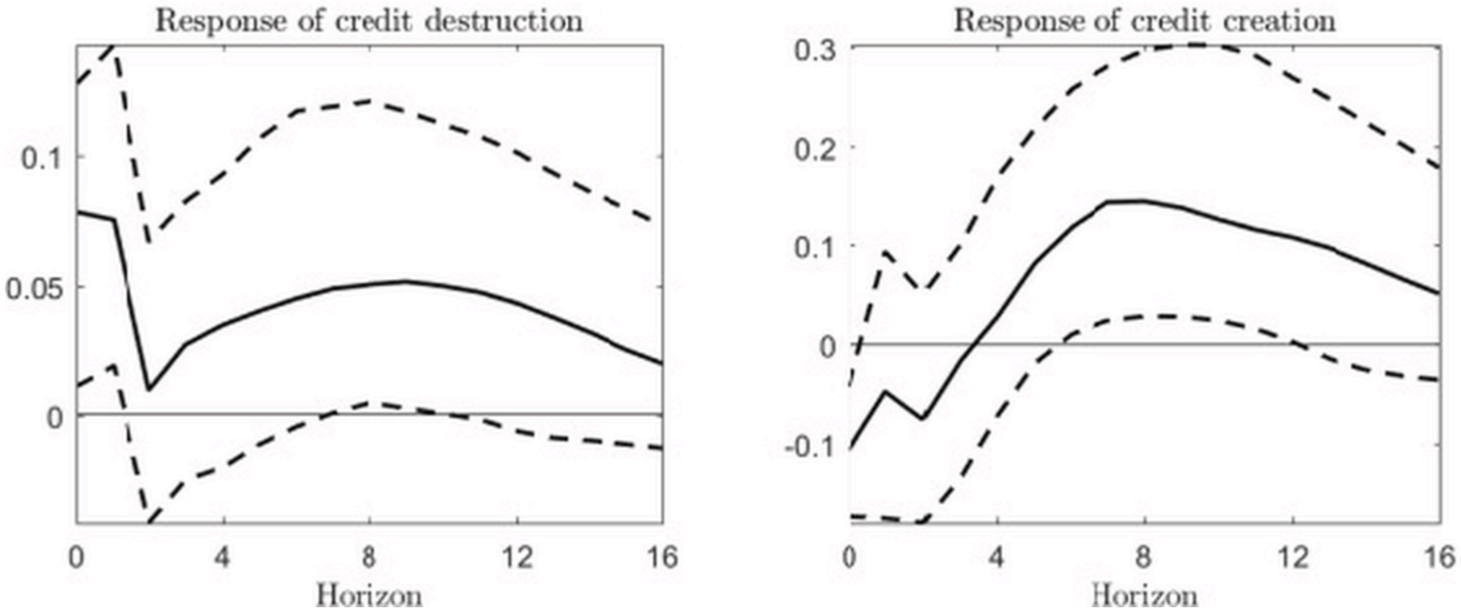
Published consumer-price and monetary-policy-rate IRFs, with their original scaling.

> At impact: the rate is 1 percentage point below its baseline.

Bianco (2021), Figure 4, p. 12

Credit creation changes sign

C39 · CREDIT DESTRUCTION AND CREATION · PERCENTAGE-POINT DEVIATIONS · 68% BANDS



Published credit-destruction and credit-creation IRFs.

Credit creation: −0.11 pp at impact → positive by Q6 → about +0.15 pp at Q8.

−100 bp policy easing · horizons in quarters · Bianco (2021), §4.2 p. 11 and Figure 4 p. 12

State the claim with its comparison attached

C40 · A RESPONSE REPORT NEEDS MORE THAN A SIGN

Under [identifying restrictions], a [size and sign] shock produces a [receiving variable] deviation from baseline of [magnitude and units] at [horizon], with [uncertainty].

> Complete the sentence for credit creation. Then name one assumption the figure cannot verify.

Keep the baseline in every interpretation

C41 · KEY TAKEAWAYS + NEXT TIME

IRF = shocked path − baseline path

Specify the shock. Trace its propagation. Report the units, assumptions and uncertainty.

The curve alone does not tell you the baseline or actual level.

> Next: FEVD asks how much shocks matter for forecast uncertainty. Module 13 also returns to system testing and cointegration/VECMs.