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SEASONAL ADJUSTMENT OF ECONOMIC TIME SERIES AND MULTIPLE REGRESSION ANALYSIS[1]

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The logical implications of certain simple consistency requirements for appraising alternative procedures for seasonal adjustment constitute the first problem considered in this paper. It is shown that any sum preserving technique of seasonal adjustment that satisfies the quite reasonable requirements of orthogonality and idempotency can be executed on the electronic computer by standard least squares regression procedures.

Problems involved in utilizing seasonally adjusted data when estimating the parameters of econometric models are examined. Extending the fundamental Frisch-Waugh theorem concerning trend and regression analysis to encompass problems of seasonality facilitates the comparison of the implications of running regressions on data subjected to prior seasonal adjustment with the effects of including dummy variables with unadjusted data. Complicated types of moving seasonal patterns that may be handled by the dummy variable procedure are considered. Although efficient estimates of the parameters of the econometric model may be obtained in appropriate circumstances when data subjected to prior seasonal adjustment by the least squares procedure are employed, there is an inherent tendency to overstate the significance of the regression coefficients; a correction procedure is suggested. When seasonally adjusted data are employed, certain special complications must be taken into account in applying Aitken's generalized least squares procedure in order to deal with autocorrelated residuals. The entire argument extends to two-stage least squares estimation of the parameters of simultaneous equation models.

1. INTRODUCTION

SEASONAL fluctuations in economic time series present problems for both the business analyst and the econometrician. In this paper we examine first the logical implications of certain simple consistency requirements that might reasonably be applied in appraising alternative procedures for seasonal adjustment. Then we consider in terms of the consistency requirements certain inherent advantages of the least squares approach to the task of seasonal adjustment. Problems encountered when seasonally adjusted data are utilized in regression analysis for purposes of parameter estimation and hypothesis testing are explored in the concluding section of the paper.

2. CRITERIA FOR APPRAISING TECHNIQUES OF SEASONAL ADJUSTMENT

Rather than evaluate alternative techniques for seasonal adjustment on the basis of a subjective appraisal of the quality of the results obtained in the experimental application of various procedures to actual or artificially constructed time series, we shall analyze the implications of a set of explicitly

[1] I am indebted to Leopold A. Sveikauskas and to my friends at the Cowles Foundation for Research in Economics at Yale University for helpful comments and constructive criticism. My colleague, Jack Muth, who was kind enough to subject the entire manuscript to careful scrutiny, unearthed innumerable errors and suggested a number of modifications improving the development of the argument. This research was undertaken during the tenure of a National Science Foundation Research grant.

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stated requirements that any ideal technique of seasonal adjustment might reasonably be expected to satisfy. An analogous approach was followed by Irving Fisher in his defense of the ideal index number.

Property I: An Adjustment Procedure is said to PRESERVE SUMS if and only if

$$x_t^a + y_t^a = (x_t + y_t)^a \qquad \text{for all } t, \tag{2.1}$$

where x_t and y_t are the original observations on any *pair of time series and x_t^a and y_t^a are the adjusted observations.*

No discrepancies in aggregation will be generated if component series rather than totals are subjected to deseasonalization with a procedure satisfying this requirement.[2] When data on unemployment is subjected to seasonal adjustment with a sum preserving procedure, precisely the same figures will be obtained as would have been provided by subtracting seasonally adjusted employment from the adjusted labor force series.[3]

Property II: An Adjustment Procedure PRESERVES PRODUCTS if and only if

$$x_t^a y_t^a = (x_t y_t)^a \qquad \text{for any two time series } x_t \text{ and } y_t. \tag{2.2}$$

The ratio of seasonally adjusted unemployment to seasonally adjusted labor force would be identical to the time series obtained by deseasonalizing the raw unemployment ratio if the adjustment procedure preserved products. With a procedure satisfying this consistency requirement, it is immaterial whether one deflates raw value series by an unadjusted price index and then subjects the deflated value series to seasonal adjustment or, alternatively, adjusts the quantity and price series in advance of deflation.[4]

Theorem 2.1: If an adjustment procedure preserves both sums and products, then it is trivial in the sense that for each t either:

$$x_t^a = x_t \text{ or } x_t^a = 0. \tag{2.3}$$

This theorem, proved in the appendix, is disturbing, for it suggests that two quite simple criteria rule out the possibility of a generally acceptable "ideal" technique for adjusting economic time series. Since there exists no nontrivial technique for seasonal adjustment that preserves both sums and products, an adjustment procedure that preserves the definitional relationship between employment, unemployment, and the size of the labor force, cannot be expected to yield an adjusted unemployment rate equal to the ratio of adjusted unemployment to adjusted labor force. While it should be recognized

[2] An interesting comparison of the effects of the direct adjustment of aggregates with the sum of adjusted components is presented by D. J. Daly [9].

[3] Professor Samuelson pointed out in a letter to the editor of the *Sunday New York Times*, November 12, 1961, that official seasonally adjusted time series do not satisfy this consistency requirement; he emphasized that when one examines the movement of "residual adjusted employment," defined as the excess of adjusted labor force over adjusted employment, the experience of the early months of the Kennedy administration appears much more satisfactory than it does in terms of official figures. John A. Brittain [4, 5] has presented detailed comparisons of the residual unemployment concept with the official series.

[4] I am indebted to Thomas M. Stanback Jr. for first bringing this problem to my attention.

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that if we are given two particular time series to compare it might be possible to construct a non-trivial adjustment procedure that would preserve both sums and products, the numerous difficulties encountered in the seasonal adjustment of unemployment data suggest that such efforts might well prove unsuccessful when with the passage of time new observations accumulated.

Now consider three additional properties.

Property III: An Adjustment Procedure is ORTHOGONAL if for any time series

$$\sum_t (x_t - x_t^a)x_t^a = 0. \tag{2.4}$$

How can a nonorthogonal seasonal adjustment procedure be regarded as satisfactory? After all, if such a procedure correctly defines the seasonal movement, the fact that the seasonal correction terms are correlated with the adjusted series implies that some seasonality remains in the data.[5]

Property IV: The Adjustment Procedure is IDEMPOTENT if

$$(x_t^a)^a = x_t^a \qquad \text{for all } t. \tag{2.5}$$

If a seasonally adjusted time series is perturbed when subjected once more to the seasonal adjustment process, we must conclude that the technique of adjustment is deficient in that it either fails to filter out all seasonal movements or else distorts series already free of seasonal fluctuations.[6]

Property V: For a SYMMETRIC Adjustment Procedure,

$$\frac{\partial x_t^a}{\partial x_{t'}} = \frac{\partial x_{t'}^a}{\partial x_t} \qquad \text{for all } t \text{ and } t'. \tag{2.6}$$

An increase in one of the observations of an unadjusted time series may be expected to affect a number of components of the time series obtained by seasonally adjusting the perturbed series. While it is no doubt reasonable in many applications to require that such effects be symmetric, it should be recognized that asymmetry does have certain advantages. In order to avoid any suggestion that the evidence is being manipulated, it may be essential to announce in advance the seasonal correction factors that will be utilized in adjusting unemployment data and other important indicators of current business conditions. Furthermore, it is embarrassing to find that seasonally adjusted figures for earlier periods are subject to marked revision as a result of updating preliminary figures for recent months.[7]

[5] Whenever one is primarily interested in product rather than sum preservation it is more convenient to state this property and the two to follow in terms of the logarithms of the original observations.

[6] It is interesting to note that the Bureau of Labor Statistics' technique of subjecting unemployment data to repeated processing by the ratio to moving average procedure achieves this desirable property by iteration. For a description see Raff and Stein [29].

[7] Seasonal patterns obtained in application of ratio-to-moving average procedures often are subject to considerable revision when additional observations become available. Mort S. Raff and Robert Stein [29, pp. 825–6] report that " . . . changes of up to 7 per cent in several of the monthly factors (were required) when 1958 data were added to the 1947–57 unemployment series. The unreasonableness of such large changes was recognized by an interagency committee, which decided to continue using the old seasonal factors until stronger evidence became available about changes in seasonal pattern." The difficulties have continued. New correction factors published in the February,

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The various seasonal adjustment properties that we have enumerated are not independent.

Theorem 2.2: Consider the idempotency, orthogonality, and symmetry properties. One can find examples of sum preserving adjustment procedures possessing any one of these three properties but not the other two. On the other hand, any sum preserving procedure that possesses two of these properties necessarily satisfies the third.

This theorem facilitates the evaluation of seasonal adjustment techniques. For example, a constant seasonal pattern is sometimes filtered out by equalizing monthly means through the subtraction of an appropriate correction term.[8] Since this procedure is obviously sum preserving, idempotent, and symmetric, it must necessarily yield correction terms that are uncorrelated with the adjusted time series. Of course, this is but one procedure possessing these various properties. Next we consider a convenient categorization of all sum preserving, orthogonal procedures possessing the idempotency property.

3. LEAST SQUARES SEASONAL ADJUSTMENT

Although least squares procedures for seasonal adjustment have been considered in articles appearing over the past twenty-five years by Arne Fisher [12], Horst Mendershausen [28], Dudley Cowden [8], A. Hald [18], L. Hurwicz [20], H. Eisenpress [11], Richard Stone [32], Julius Shiskin and Eisenpress [31, p. 441], and John Frechtling [13], the official adjustment of economic time series by government agencies is usually achieved through application of some variant of the ratio-to-moving average approach. Perhaps the least squares procedure will gain more general acceptance once its advantages in terms of the properties examined in this paper are recognized.

Theorem 3.1: Any sum preserving adjustment procedure that is orthogonal and idempotent (and hence symmetric) can be executed by regressing the unadjusted time series upon an appropriate set of explanatory variables; conversely, the residuals obtained through the regression of the data upon an appropriate set of explanatory variables constitutes an adjusted time series satisfying the requirements of sum preservation, idempotency, orthogonality, and symmetry.

The type of seasonal disturbance filtered out by the least squares approach is determined by the specification of the explanatory variables. This proposition, which follows at once from the fact that the observed residuals obtained from regression analysis are necessarily uncorrelated with the explanatory variables, deserves illustration. For the special case of a stable seasonal pattern that does not change from one year to the next, regress x_{ym}, the observation for season m of the year y, upon a set of dummy variables:

$$x_{ym} = \sum_{i=1}^{k} b_i s_{ymi} + e_{ym}, \tag{3.1}$$

1962, *Monthly Labor Review* reveal further revisions of fair magnitude of correction factors calculated with data through September, 1961, although the differences were probably no larger than should be expected in any case from sampling error.

[8] If x_{my} is the unadjusted observation of the mth month of the yth year, calculate

$$x^{a}_{my} = x_{my} + \left[\frac{\sum\sum}{ym} x_{my}/t\right] - \frac{12\sum_{y} x_{my}}{t},$$

where t is the number of observations.

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where k is the number of seasons in a year

$$s_{ymi} = \begin{cases} 1 & \text{if } i = m \\ 0 & \text{otherwise.} \end{cases}$$

The mean of the original series, $\bar{x}$, may be added to the observed residuals in order to yield $x^a_{ym} = e_{ym} + \bar{x}$ as the adjusted observation.[9] To allow for trend and seasonality simultaneously, regress:

$$x_{ym} = \sum_{i=1}^{k} b_i s_{ymi} + \sum_{i=1}^{n} c_i t^i + e_{ym}, \quad \text{where} \quad t = 12y + m, \tag{3.2}$$

a procedure originally described by Arne Fisher [12] and applied by A. Hald [18] in an analysis of interwar percentage unemployment in Denmark. In order to allow for a moving seasonal, follow Dudley Cowden's suggestion [8] that we regress:

$$x_{ym} = \sum_{m} s_{ym}(a_{0m} + a_{1m}t + \cdots + a_{dm}t^n) + e_{ym},$$
$$\text{where } s_{ym} = \begin{cases} 1 & \text{in season } m \\ 0 & \text{otherwise.}^{10} \end{cases} \tag{3.3}$$

If the seasonal pattern may be assumed to display a certain amount of continuity from one season to the next, as with daily or weekly data, it may be possible to economize on the use of explanatory variables by performing the following regression:

$$\begin{aligned} x_t &= a_0 + \sum_{i=1}^{n} a_i \sin(\gamma_i t + b_i) + e_t \\ &= a_0 + \sum_{i=1}^{n} \{a_i \cos b_i[\sin(\gamma_i t)] + a_i \sin b_i[\cos \gamma_i t]\} + e_t, \end{aligned} \tag{3.4}$$

[9] It is readily seen that the x^a_{my} are identical to the set of seasonally adjusted data that would be obtained by the equalization of monthly means procedure. After all, the data may be sorted by months and k separate regressions run without affecting the b_i. But each b_i is equal to the seasonal mean, for the second moment is at a minimum about the mean and as a consequence the procedure is equivalent to that described in footnote 8.

[10] While this approach does not segregate moving seasonality from trend, that distinction is necessarily befuddled once the possibility of a moving seasonal pattern is introduced unless the explanatory variables can be partitioned explicitly into trend and seasonal categories. A two-stage procedure of first removing trend and then seasonality, as with the most elementary version of the ratio-to-moving average procedure, is usually sensitive to the arbitrary decision to delete trend rather than seasonality first. Sometimes trend is segregated from seasonality by blowing up preliminary deseasonalized data so as to avoid any discrepancy between the annual totals of the raw and adjusted time series. But if we circumvent the arbitrary calendar convention by making the totals over any consecutive twelve-month period be undistorted, we are forced to exclude from consideration any adjustment procedure that allows for a flexible seasonal pattern. To see this, suppose that the requirement

$$\sum_{t=k}^{k+12} x^a_t = \sum_{t-k}^{k+12} x_t$$

is satisfied for all k; then

$$\sum_{t-k}^{k+12} x_t - \sum_{t-k+1}^{k+13} x_t = \sum_{t-k}^{k+12} x^a_t - \sum_{t-k+1}^{k+13} x^a,$$

or

$$x_k - x_{k+13} = x^a_k - x^a_{k+13}.$$

Therefore,

$$x_k - x^a_k = x_{k+13} - x^a_{k+13},$$

proving that the January correction factor has not changed. Iteration in this way proves that each season's correction factors must be a constant.

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where

$$\gamma_i = \frac{2\pi i}{k}$$

k = number of seasons in a year,
$2n < k$,
$a_i \cos b_i$ and $a_i \sin b_i$ are the regression coefficients;

the expressions in brackets are the explanatory variables.

With any of these approaches, it is possible to add dummy variables to indicate the occurrence of Easter or such irregular events as strikes. One may also filter out the influence of climate by including explicit measures of precipitation and temperature; in a study of fuel oil sales an index of "degree days" may be introduced. Of course, the length of the time series imposes a restriction on the variety of forces that may be included as explicit generators of seasonal variation.

While the least squares procedure possesses considerable flexibility, it also makes explicit the element of judgment inherent in any process of time series decomposition. Statistical inference is facilitated when the least squares approach is employed. R. L. Anderson [1] has explained how a simple analysis of variance procedure may be applied in order to test for seasonality of the type suggested by equation (3.1). Richard Stone [32], in an illustration with quarterly United Kingdom consumption data of the application of covariance analysis to the problem of testing for types of seasonal movements, rejected a model of form (3.3) in favor of the simpler system (3.2) involving a constant seasonal pattern. Problems of statistical inference will be discussed within a more general context in the next section of this paper.

4. REGRESSION ANALYSIS WITH SEASONALLY ADJUSTED DATA

When justification is offered for the popular ratio to moving average procedure for seasonal adjustment, it is usually in terms of the assumption that the time series of interest is the product of a trend and cyclical forces, a moving seasonal, and an erratic disturbance;[11] the problem of seasonal adjustment is that of filtering out the seasonal factor without seriously distorting the other elements generating the observed data.[12] In contrast, the business analyst who utilizes adjusted time series is hardly likely to regard the data as having been generated in this way. He is generally all too aware of the multitude of interacting economic forces determining the movements of the time series in question. The trend, seasonal, irregular trichotomy underlying the traditional theory of

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[11] Kendall [23, p. 370] describes trend as a "smooth broad motion of the system over a long term of years," and seasonality as "a fluctuation imposed on the series by a cyclic phenomenon external to the main body of causal influences at work upon it."

[12] Most writers substitute illustrative examples of the effects of eliminating seasonality for an explicit statement of the process by which the data is generated. In contrast, Hannan [19] provides a precise statement of the problem. Bongard [3] considers explicitly the problem of finding that centered 19-term moving average that will (a) eliminate a constant seasonal pattern, (b) faithfully reproduce a third degree curve, and (c) minimize the variance of the residuals.

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time series decomposition is alien to any attempt at a causal explanation of the operations of the economy.

Suppose that the variable of interest, y_t, is generated by a linear model of a form familiar to both the economic theorist and the econometrician.

$$y_t = \sum_i \beta_i x_{it} + s_t + \epsilon_t, \qquad \mathcal{E}(\epsilon_t) = 0. \tag{4.1}$$

Here x_{it} is the level of the ith explanatory variable at time t, ϵ_t is an unobserved random error term representing the effect of omitted variables or unobservational error, and β_i are unknown parameters. Although the unobserved systematic seasonal disturbances s_t may be assumed as a special case to change from one season to the next but to take on the same value in any given season each year,[13] the argument that follows concerning the problem of estimating the parameters of (4.1) can be generalized in order to take much more complicated types of seasonal movements into account, such as those considered in section 3 above.

If we subject the variable of interest, y_t, to seasonal adjustment with a procedure that preserves sums and, in addition, is capable of filtering out the seasonal disturbance s_t (i.e., $s_t^a = 0$ for all t) we shall obtain:

$$y_t^a = \sum_i \beta_i x_{it}^a + \epsilon_t^a. \tag{4.2}$$

Because examination of the adjusted series reveals movements generated by atypical changes in the nonseasonal economic forces x_{it}^a, seasonal adjustment may aid the business analyst in his task of appraising current economic conditions. It is by no means obvious that the econometrician interested in testing hypotheses or in estimating the parameters of (4.1) will find the seasonally adjusted time series most appropriate for his purposes. Current econometric studies reveal a lack of concensus concerning the most suitable technique for handling seasonal movements. While time series adjusted by the ratio to moving average procedure have been employed in studies by Barger and Klein [2], by Colin Clark [6], by Duesenberry, Eckstein, and Fromm [10], the Klein, Ball, Hazlewood, and Vandome econometric study of the United Kingdom [25] constitutes but one example of the introduction of seasonal dummy variables in an attempt to net out the effects of seasonality present when working with unadjusted time series.[14]

It proves convenient to adopt matrix notation in demonstrating the circumstances in which the application of least squares to seasonally adjusted data will yield unbiased parameter estimates. We write our linear model as:

$$Y = X\beta + S\alpha + \epsilon, \qquad \mathcal{E}(\epsilon) = 0, \quad \text{where} \tag{4.1*}$$

[13] So that $s_{t+12} = s_t$ with monthly data. As we shall see, this is precisely the circumstances in which the practice of including one dummy variable for each season in regression analysis is appropriate.

[14] Although the econometrician is usually free to choose between these two alternative approaches, the National Income Division, Department of Commerce, publishes only seasonally adjusted quarterly data for certain components of the national income accounts. In a study of the behavior of prices, inventory, and production in the shoe, leather, hide sequence, Kalman Cohen [7, pp. 89–92] attempted to reintroduce estimated seasonal movements into time series available to him only in adjusted form.

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Y is a t component column vector,
X is a $t \times k$ matrix of explanatory variables,
ϵ is a t component column vector of disturbances,
$S = D\alpha$, the unknown $t \times 1$ vector of seasonal disturbances, belongs to a particular family of possible seasonal movements characterized by the $t \times d$ matrix D,
α is an unknown $d \times 1$ vector.

Now the application of least squares to calculate the $k \times 1$ vector b of regression coefficients that minimizes the sum of squared residuals $e'e$, where $e = Y_a - bX_a b$ provides:

$$b = (X_a' X_a)^{-1} X_a' Y_a = \beta + (X_a' X_a)^{-1} X_a' (Y_a - X_a \beta). \tag{4.3}$$

Unless the expected value of the second term on the right of the last equality vanishes, the application of least squares to seasonally adjusted data will yield biased estimates of the parameter vector β. Suppose that the technique of seasonal adjustment preserves sums; then there exists a $t \times t$ matrix A such that $Y_a = AY = AX\beta + AD\alpha + A\epsilon$.[15] If, in addition, the adjustment procedure is capable of annihilating S in the sense that $AD = 0$, we have $Y_a - X_a\beta = A\epsilon$. When the adjustment procedure has these two properties, equation (4.3) reduces to:

$$b = \beta + (X_a' X_a)^{-1} X_a' A\epsilon. \tag{4.4}$$

Clearly, b will be an unbiased estimator of β if the X are fixed from sample to sample.

If our data have been seasonally adjusted by the least squares procedure with the matrix D as the set of explanatory variables, the adjustment matrix $A = I - D(D'D)^{-1}D'$ will clearly be capable of annihilating S, as required for unbiased parameter estimation with seasonally adjusted data. Of course, if H is any $t \times t$ matrix, the adjustment matrix HA will constitute a sum preserving procedure capable of filtering out $D\alpha$. While least squares adjustment is thus seen to be but one member of a larger family of procedures permitting us to obtain unbiased estimates of the parameters of (4.1) with seasonally adjusted data, an extension of a fundamental theorem of Ragnar Frisch and Frederick Waugh [15] concerning regression analysis and trend assists in specifying conditions in which least squares seasonal adjustment is preferred. Frisch and Waugh established that the least squares removal of linear trend from individual time series in advance of regression yields parameter estimates identical to those that would have been obtained with unadjusted data if time had been included as an additional explanatory variable. Tintner [35, p. 304] extended their argument to encompass polynomial trends. More generally,

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[15] In order for a seasonal adjustment process to constitute a linear transformation, we must have by definition [34, p. 32] for every vectors X and Y of t observations and scalar λ both (1) $X_a + Y_a = (X+Y)_a$ (sum preservation) and (2) $\lambda X_a = (\lambda X)_a$. While (2) seems a reasonable requirement to impose upon processes of seasonal adjustment, and indeed is satisfied by all common procedures, it is actually implied by (1) when attention is restricted to the field of rational numbers, as is the case when seasonal adjustment procedures are applied to published data.

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Theorem 4.1: Consider the following alternative regression equations, where the subscript α indicates that the data have been adjusted by the least squares procedure with D as the matrix of explanatory variables:

1. $Y = Xb_1 + Da_1 + e_1$
2. $Y_\alpha = X_\alpha b_2 + e_2$
3. $Y = Xb_3 + e_3$
4. $Y = X_\alpha b_4 + e_4$
5. $Y_\alpha = Xb_5 + e_5$
6. $Y = X_\alpha b_6 + Da_6 + e_6$
7. $Y_\alpha = Xb_7 + Da_7 + e_7$.

Then if the rank of $[X; D]$ is $k+d<t$,[16] *the regression coefficients obtained by application of least squares satisfy:*

$$\begin{aligned} b_1 &= b_2 = b_4 = b_6 = b_7 \\ &= b_3 + (X'X)^{-1}X'[(X - X_\alpha)b_1 - (Y - Y_\alpha)] \\ &= b_5 - (X'X)^{-1}X'(X_\alpha - X)b_1, \\ a_6 &= a_1 + \overset{0}{a_x} b_1, \qquad \text{where } \overset{0}{a_x} = (D'D)^{-1}D'X \\ a_7 &= a_1 - \overset{0}{a_y}, \qquad \text{where } \overset{0}{a_y} = (D'D)^{-1}D'Y. \end{aligned}$$

If e_i denotes the observed residuals of the ith regression, then

$$e_1 = e_2 = e_4 - (Y - Y_\alpha) = e_6 = e_7.$$

The variance of the residuals satisfy:

$$S_1^2 = \frac{e_1'e_1}{n} = S_2^2 \leq \frac{e_4'e_4}{n} = S_1^2 + \frac{1}{n}(Y - Y_\alpha)'(Y - Y_\alpha)$$

$$S_1^2 \leq \frac{e_5'e_5}{n}.$$

The multiple correlation coefficients satisfy:

$$\begin{aligned} R_5 \leq R_7 = R_2 \leq R_1 = R_6 \\ R_4 \leq R_6 \\ R_3 \leq R_1. \end{aligned}$$

The proof of the theorem, which is related to suggestions of Rao [30, pp. 118–21] and discussions of residual analysis by Freund, Vail, Clunies-Ross [14] and by Goldberger and Jochems [16], is relegated to the appendix.

Five alternative least squares regression procedures yield identical estimates of the parameters of (4.1). The identity $b_1 = b_2$ reveals that inclusion of the matrix of seasonal dummy variables in the regression analysis is equivalent to

[16] If the matrix D of dummy variables is constructed carelessly, this rank condition may be violated. Caution is required when a column of X consists only of ones in order that the corresponding element of the vector b of regression coefficients constitutes the intercept; for example, when a constant seasonal is being introduced with quarterly data, D should have only three columns. For a discussion of dummy variables see Suits [33].

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working with least squares adjusted time series.[17] The identity $b_2 = b_4$ reveals that it is immaterial whether the dependent variable is adjusted or not, provided the explanatory variables have been seasonally corrected.

The same residuals will be observed regardless of whether regression procedure 1, 2, 6, or 7 is applied. While the same unexplained variance will be obtained with any one of these five alternative regression procedures, the magnitude of the variance of the dependent variable depends upon whether one works with adjusted or unadjusted data. As a consequence, deseasonalizing the dependent variable in advance of running the regression, as in equation (2) or (7), worsens appearances by reducing the size of the multiple correlation coefficient.

With problems of hypothesis testing and confidence interval construction the choice between alternative regression procedures becomes crucial. Although procedures 1, 2, 6, and 7 yield the same observed residuals and have the same inverse relevant to the estimation of the standard errors of the components of β,[18] considerable care must be exercised in making appropriate adjustments for the loss of degrees of freedom that occurs when deseasonalized data is employed. Utilizing deseasonalized data with regression 2 suggests that there are $t-k$ degrees of freedom; procedures 1, 6, and 7, on the other hand, imply that there are only $t-k-d$ degrees of freedom. Clearly, at least one of the approaches is misleading! The root of the problem may be perceived by premultiplying (4.1*) by A, the adjustment matrix, in order to obtain an expression for the adjusted dependent variable:

$$Y_a = X_a + A\epsilon, \tag{4.4}$$

where we suppose $\mathcal{E}(\epsilon) = 0$ and $\mathcal{E}(\epsilon'\epsilon) = \sigma^2 I$; later, problems created by autocorrelated error terms will be considered. Now the disturbance terms in this last equation, $A\epsilon$, are necessarily free of any seasonal pattern; because of this artificial restriction, they cannot be truly independent.[19]

Fortunately, the loss of degrees of freedom may be appropriately taken into account, at least when the data have been adjusted by the least squares procedure described in section 3 above. While the point estimates of the regression coefficients are unaffected, their estimated standard errors are sensitive to the way in which the degrees of freedom are tabulated. If the calculations have

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[17] Provided, of course, that the same matrix D is utilized in constructing the adjustment matrix A as is utilized in the dummy variable regression. It should be noted that in most applications of the dummy variable procedure D takes a particularly simple form implying an unchanging seasonal pattern. If in fact the D that is utilized in the dummy variable procedure is not capable of annihilating S, the erroneous results obtained may be interpreted as the effect of omitting variables that are correlated with other explanatory variables, a difficulty discussed by Wold [37, p. 37]. Lawrence Klein [24, pp. 313-7] discusses the use of dummy variables in the case of an unchanging seasonal pattern and also explains how one may proceed when certain components of β are presumed to be subject to seasonality, a complication that cannot be readily taken into account by any additive procedure of seasonal adjustment.

[18] It is easily verified that $(X_a'X_a)^{-1}$, the inverse of the moment matrix of the second regression, constitutes the upper left-hand block of the inverse of the first regression's moment matrix. Let M be the moment matrix of the first regression, and compute $M^{-1} = (M^{-1}P^{-1})P = (PM)^{-1}P$, where

$$P = \begin{bmatrix} I & -X'D(D'D)^{-1} \\ 0 & (D'D)^{-1} \end{bmatrix}.$$

[19] If A is capable of filtering out some form of seasonal movement, then for some $S = D\alpha \neq 0$ we must have $AS = 0$. The rank of A is then $t-d$, where $d \geq 1$ is the dimension of the type of seasonal movement filtered out by A. Hence the disturbances of (4.4) cannot span a space of dimension greater than $t-d$.

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been performed without considering the effects of seasonal adjustment, $t-k$ rather than $t-k-d$ degrees of freedom will have been employed in deriving the standard errors of the regression coefficients; they should be multiplied by the following correction factor:

$$\sqrt{\frac{t-k}{t-k-d}}. \tag{4.5}$$

The same correction factor should also be employed if $t-k$ degrees of freedom were mistakenly assumed in calculating the unbiased estimate of σ_ϵ. Corrected R^2 also requires adjustment.[20]

In spite of the explicit warning in Lawrence Klein's *Textbook of Econometrics* [24, p. 321], the loss of degrees of freedom occasioned by the employment of data subjected to prior adjustment for trend or seasonal movement is commonly neglected in reporting econometric results. Although the task of calculating the precise number of degrees of freedom lost when data have been subjusted to adjustment by the ratio to moving average procedure might well prove intractable, it may well be more appropriate to employ factor (4.5) as an approximation than to neglect the problem entirely. Since the ratio to moving average procedure allows for a shifting seasonal pattern, one might well choose $d=3m-1$, where m is the number of seasons.[21] For monthly data, neglect of the effects of seasonal adjustment may overstate the actual number of degrees of freedom by 35; for quarterly data, 11 too many degrees of freedom may have been attributed to the data. The effect is to overstate the significance of regression coefficients.[22]

A convenient assumption, frequently invoked in regression analysis, is that the disturbances are distributed with constant variance and are free of autocorrelation; more precisely, it is required that $\mathcal{E}(\epsilon\epsilon')=I\sigma^2$.[23] At least if the X are fixed from sample to sample, this assures that the application of the method of least squares to regression 1 will yield best linear unbiased estimates of the

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[20] In adjusting R^2 for loss of degrees of freedom the customary formulas lead to a corrected coefficient of determination equal to the ratio of the unbiased estimate of the explained variance to an unbiased estimate of the unexplained variance. The corresponding formula when the data has been seasonally adjusted is

$$\bar{R}_c^2 = \left[1 - (1 - R^2)\left(\frac{t-d}{t-k-d}\right)\right].$$

[21] Even this is a conservative figure, for it does not allow much flexibility to the moving seasonal. See equation (3.3). When an intercept is not included in the regression, we should let $d=3m$.

[22] In reporting regressions obtained in a study of inventory investment, I failed to consider the costs in terms of degrees of freedom imposed by the fact that only seasonally adjusted data were available [27, p. 300]. There were 29 quarterly observations and six coefficients in certain regression equations. Formula (4.5) yield a correction factor of

$$\sqrt{\frac{29-6-11}{29-6}} = .72.$$

Now if we had observed $b_i/s_{bi}=2.9$, the coefficient would have appeared to be significant at the 0.01 level with $n-k=23$ degrees of freedom. In contrast, application of the correction factor reduces the ratio to 2.09, which is not significant at even the 0.05 level with $n-k-d=12$ degrees of freedom.

[23] While it may be reasonable in many applications to assume that ϵ is distributed independently of D, a possible exception may arise with a "4-8-4" sampling pattern sometimes utilized in data collection; households that enter the sample in January, say, are also interviewed in February, March, and April, and again in the corresponding four months of the following year.

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parameters of $Y_t = X\beta + D\alpha + \epsilon$. If, in addition, the residuals are normally distributed the application of least squares yields maximum likelihood estimates of β. For any sample, however, precisely the same estimates of β will be obtained with regression 2 as with 1, provided the data have been adjusted with the least squares procedure. Consequently, best linear unbiased estimates of β are obtained when the regression is performed upon data adjusted with the least squares technique, provided the independence condition on the ϵ_t is satisfied. This is a distinctive property of the least squares adjustment procedure. If the explanatory variables are adjusted with any alternative to the least squares technique, best linear unbiased estimates will not be obtained.

Further complications arise with regard to the tasks of estimation and hypothesis testing when the disturbances of (4.1) are autocorrelated.[24] Aitken's generalized least squares analysis suggests that best linear unbiased estimators could be obtained by the application of least squares to the transformed equation

$$HY = HX\beta + HD\alpha + H\epsilon, \tag{4.6}$$

where the transformation matrix H has the properties:

$$H\Omega H' = I \quad \text{and} \quad H'H = \Omega^{-1},$$

where $\Omega = \mathcal{E}(\epsilon\epsilon')^{-1}$.

By the generalized Frisch-Waugh theorem, an identical estimate of β will be obtained by the application of least squares to

$$\tilde{A}Y = \tilde{A}X\beta + \tilde{A}\epsilon, \tag{4.7}$$

where

$$\tilde{A} = [I - HD(D'H'HD)^{-1}D'H']H.$$

This adjustment matrix annihilates D, and (4.7) is the same expression as would be obtained by adjusting (4.6) by the least squares adjustment matrix that filters out HD. Since precisely the same residuals are observed with regressions performed on data adjusted by the least squares matrix A as are provided by unadjusted data with dummy variables explicitly included, the same information is available for estimating Ω. Nevertheless, the customary practice of applying an estimated H directly to seasonally adjusted data is *not* appropriate; regressing $HAY = HAX\beta + HA\epsilon$ will not in general yield the efficient estimate of β provided by (4.6) or (4.7).[25]

The contrast between regressions 1 and 3 also deserves mention, for it reveals conditions under which best linear unbiased estimates of β may be obtained with unadjusted data without dummy variables. Although both the dependent and independent variables may be subject to considerable seasonal

[24] L. Hurwicz has argued [20] that the shortening of the observation period contributes to more severe autocorrelation problems.

[25] Observe, however, that at least in principle data already adjusted by A may be transformed into the $\tilde{A}$ adjusted form, and this despite the fact that the singularity of A means that adjusted data cannot be transformed back into the original form. Thrall and Tornheim [34, p. 99] demonstrate, Theorem 3.7G, that if $\tilde{A}$ and A are two $t \times t$ matrices and if $\tilde{A}D = 0$ if and only if $AD = 0$, then there exists a non-singular matrix P such that $PA = \tilde{A}$. If A were not singular, $AD = 0$ implies $D = 0$.

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movement, it is possible that the seasonality in the dependent variable is entirely the consequence of seasonal influences acting indirectly through the explanatory variables. In this case, model (4.1*) is correctly formulated as:

$$Y = X\beta + \epsilon, \tag{4.8}$$

and the regression is appropriately performed with unadjusted data. But if the seasonality in the dependent variable really is generated via the explanatory variable, if $(X-X_a)\beta = Y-Y_a$, the discrepancy between the estimates obtained with this procedure and alternatives 1 and 2 arises solely from sampling error and does not lead to bias. Degrees of freedom equal to the number of columns of D are lost when the regression is performed with the adjusted data.[26] It would be better, in these circumstances, to work with unadjusted rather than adjusted time series.

In conclusion, it must be conceded that nothing in this paper has ruled out the possibility that there exists some alternative class of stochastic model for which the ratio to moving average procedure is preferable; but surely it behooves any econometrician who would use data adjusted by this more popular technique to demonstrate that this is indeed an appropriate approach to the problem of estimating the parameters of his model. Note, too, that although the argument of this paper has been developed only for linear models, it generalizes immediately to multiplicative models linear in the logs, the case in which least squares seasonal adjustment preserves products rather than sums. Furthermore, the analysis admits an obvious extension to a simultaneous equation framework; in particular, it is apparent that the two-stage least squares procedure may be applied either with data subjected to prior adjustment by the least squares procedure or with seasonal dummy variables included explicitly as exogenous terms in the first stage regression equations.

APPENDIX

PROOF OF THEOREM 2.1:

Property I implies that the adjustment process constitutes a linear transformation[27] and may therefore be represented by the expression:

$$x_t^a = \sum_\tau a_{t\tau} x_\tau.$$

If, in addition, the procedure satisfies the multiplicative requirement we must have:

$$x_t^a y_t^a = \left(\sum_\tau a_{t\tau} x_\tau\right)\left(\sum_\tau a_{t\tau} y_\tau\right) = (x_t y_t)^a = \sum_\tau a_{t\tau} x_\tau y_\tau,$$

[26] An interesting application is provided by the conjecture of Jack Johnston [21] that if manufacturers attempt to smooth the impact of seasonal variation in sales upon production, the effect can be adequately taken into account by including constant seasonal dummy variables in the regression (in this case D has a particularly simple form); the magnitude of the dummy variables coefficient indicates the extent to which planned inventories fluctuate in the attempt to stabilize production. The conjecture that manufacturers do smooth production in this way can be subjected to empirical test by determining whether the addition of seasonal dummy variables leads to a significant reduction in the unexplained variance.

[27] See footnote 15.

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an equality that can hold for *every* two arbitrary time series only if $a_{tr}=0$ whenever $t\neq r$, and $a_{tt}=1$ or zero.

PROOF OF THEOREM 2.2:

Observe that the fact that a sum preserving seasonal adjustment process constitutes a linear transformation implies that there exists a $t\times t$ matrix A such that $X_a=AX$, where $X=\text{col}(x_t)$ and $X_a=\text{col}(x_t^a)$.

The following three matrices constitute sum preserving adjustment procedures that respectively satisfy only Properties III, IV, and V.[28]

$$\begin{bmatrix} \frac{1}{3} & \frac{-\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & \frac{1}{3} \end{bmatrix}, \quad \begin{bmatrix} .5 & 1 \\ .25 & .5 \end{bmatrix}, \quad [2].$$

Since IV and V imply that $A'A=AA=A$, $(I-A)'A=0$; hence

$$\sum_t (x_t - x_t^a)x_t^a = (X - X_a)'X_a = X'(I - A)'AX = 0,$$

which is III.

Note Properties I and III imply that

$$\sum_t (x_t - x_t^a)x_t^a = X'(I - A)'AX = 0$$

for all X. Therefore $Z=(I-A')A=A-A'A=-Z'$, a skew symmetric matrix [34, p. 151]. Hence $A'A=\frac{1}{2}(A+A')$ and $Z=\frac{1}{2}(A-A')$.

Post multiplication by A of $A'A$ then yields, if we have Property IV as well as I and III, $A'AA=A'A=\frac{1}{2}(A+A'A)$ or $A=A'A$; hence $Z=\frac{1}{2}(A-A')=0$, implying $A=A'$, which is V.

If, on the other hand, we have Property V as well as I and III, it is apparent from $Z=\frac{1}{2}(A-A')=A-A'A$ that $A=A'$ yields $Z=0$; hence $A=A'A=AA$ which is IV.

PROOF OF THEOREM 3.1:

By Properties I and IV there exists a matrix A such that for all X, $X_a=AX$ and, in addition $A(X-X_a)=0$. Hence there exists a vector α such that $X-X_a=D\alpha$, where D is a $t\times d$ matrix formed from the column vectors constituting a basis of the column kernel of A; thus $AD=0$; furthermore $d=$rank of $D=t-$rank of A, and hence the $d\times d$ matrix $D'D$ is nonsingular. Remembering that by Property V, $AD=0$ implies $D'A=0$, we note that premultiplying $X-X_a=D\alpha$ by $(D'D)^{-1}D'$ yields $(D'D)^{-1}D'(X-X_a)=(D'D)^{-1}D'(X-AX)=(D'D)^{-1}D'X=\alpha$, and it is apparent from the last equality that α constitutes the vector of regression coefficients provided by minimizing the sum of squares $X_a'X_a=(X-D\alpha)'(X-D\alpha)$. Hence $X_a=X-D\alpha$ constitutes the residuals obtained by regressing X on D. For the converse, note that the residuals of a regression

[28] I am indebted to Jack Muth for suggesting the first matrix.

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may be expressed as $X_a = X - D\alpha$, where $\alpha = (D'D)^{-1}D'X$; hence $X_a = AX$, where $A = I - D(D'D)^{-1}D$, and it is clear that $A' = A = AA$.

The residuals will sum to zero if the matrix D contains a column of ones (the corresponding regression coefficient is then the intercept) or if the space spanned by the column vectors constituting D includes this unit vector. In these circumstances, the mean of the observations is appropriately added to the residuals in order that the mean of the seasonally adjusted series will equal the mean of the original data. To show that the residuals plus the mean of the observations will have the desired properties, let $S = \text{col}\langle 1, 1, \cdots, 1\rangle$, a $t \times l$ vector; thus $S'X^a = 0$ for all X; hence $S'A = 0$. So if

$$A^* = A + \frac{1}{n}SS',$$

then $A^* = A^{*\prime}$ and

$$A^*A^* = AA + \frac{1}{n^2}SS'SS' = AA + \frac{1}{n}SS' = A^*.$$

Now

$$\frac{1}{n}S'X = \overline{X};$$

hence

$$A^*X = \left[A + S\left(\frac{1}{n}S'\right)\right]X = AX + S\overline{X}.$$

PROOF OF THEOREM 4.1:

The seven alternative regressions listed in Theorem 4.1, reduce to

$$(\alpha)\quad Y = Xb_\alpha + Da_\alpha + e_\alpha,$$
$$(\beta)\quad Y_a = X_a b_\beta + Da_\beta + e_\beta, \quad \text{or}$$
$$(\gamma)\quad Y_a = X_a b_\gamma + e_\gamma,$$

where we suppose that we are presented with seasonally adjusted observations $Y_a = Y - Da_y$ and $X_a = X - Da_x$, upon appropriate specification of a_y and a_x. Since the application of least squares to (α) selects the unique values $\hat{b}_\alpha$ and $\hat{a}_\alpha$ minimizing $e'_\alpha e_\alpha$, we can achieve this same minimum sum of squares by letting $b_\beta = \hat{b}_\alpha$ and $a_\beta = a_\alpha - a_y + a_x b_\beta$, for then $e_\alpha - e_\beta = Da_y - Da_x b_\beta - X_\alpha(b_\alpha - b_\beta) - D(a_\alpha - a_\beta) = 0$. These must be the coefficients minimizing the sum of squares of regression (β), for if some alternative set achieved a smaller sum of squares, we could do as well with regression (α), in contradiction of the fact that $\hat{b}_\alpha$ and $\hat{a}_\alpha$ minimize the sum of squares of (α). By having $a_y = 0$ and $a_x = a_x^0$, this yields $b_1 = b_6$ and $a_6 = a_1 + a_x^0 b_\beta$; with $a_y = a_y^0$ and $a_x = 0$ we have $b_1 = b_7$ and $a_7 = a_1 - a_y^0$.

In order to determine circumstances in which the application of least squares to (γ) necessarily yield precisely the same vector of regression coefficients, con-

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sider the normal equations of (β):

$$\begin{bmatrix} X_a'X_a & X_a'D \\ D'X_a & D'D \end{bmatrix}\begin{bmatrix} b_\beta \\ a_\beta \end{bmatrix} = \begin{bmatrix} X_a' \\ D' \end{bmatrix} Y_a$$

premultiplying by $[(X_a'X_a)^{-1}; 0]$ yields:

$$b_\beta + (X_a'X_a)^{-1}X_a'Da_\beta = (X_a'X_a)^{-1}X_a'Y_a = b_\gamma.$$

Now $b_\beta = b_\gamma$ if $X_a'D = 0$, which requires from Theorem 3.1 that $a_x = a_x^0$ (that the explanatory variables be adjusted by the least squares procedure); hence $b_2 = b_1$; furthermore, $X_a'Y_a = X'AAY = X'AY = X_a'Y$, implying that $b_4 = b_1$.

Premultiplying the normal equations of regression (4) by $(X'X)^{-1}$ yields

$$(X'X)^{-1}X_\alpha'X_\alpha b_4 = (X'X)^{-1}X'X_\alpha b_4 = (X'X)^{-1}X_\alpha'Y_\alpha = (X'X)^{-1}X'Y_\alpha$$

or

$$(X'X)^{-1}X'(X_\alpha - X + X)b_4 = (X'X)^{-1}X'(Y_\alpha - Y + Y).$$

Since $b_3 = (X'X)^{-1}X'Y$, we have

$$b_4 + (X'X)^{-1}X'(X_\alpha - X)b_4 = b_3 + (X'X)^{-1}X'(Y_\alpha - Y),$$

or

$$b_4 = b_3 + (X'X)^{-1}X'[(X - X_\alpha)b_4 - (Y - Y_\alpha)].$$

Since $b_5 = (X'X)^{-1}X'Y_\alpha$ we also have

$$b_4 + (X'X)^{-1}X'(X_\alpha - X)b_4 = (X'X)^{-1}X'Y_\alpha = b_5.$$

Subtracting regression 2 from 4, and remembering $b_2 = b_4$, yields

$$Y - Y_\alpha = e_4 - e_2, \quad \text{or} \quad e_4 - (Y - Y_\alpha) = e_2.$$

The restrictions on the residuals follow at once, provided one observes that $Y - Y_\alpha = Da_y$, and consequently, by regression 1, is distributed independently of $e_1 = e_4$.

The restrictions on the coefficients of multiple correlation are easily derived once it is observed that $Y_\alpha'Y_\alpha \leq Y'Y$.

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